PHY3010 – Fall 2007
October 31, 2007

<u>Hamilton's Method</u>

Steps to solution:

- Select a "proper" set of generalized coordinates and construct T and U. Apply all constraint relations[‡] (this is not required, but is safe and simple). Construct L.

- Find the conjugate momenta via:

$$p_j = \frac{\partial L}{\partial \dot{q}_j}$$

- Invert the above equation and write the generalized velocities in terms of the conjugate momenta.

- Construct the Hamiltonian from:

$$H(q_j, p_j, t) = \sum_j p_j \dot{q}_j = L(q_j, \dot{q}_j, t)$$

  by plugging in for the generalized velocities their form in terms of the momenta. It is critical that you express the Hamiltonian in terms of the coordinates and their momenta (and possibly time).

  **NOTE*: You could write $H=T+U=E$ if and only if:

  1. The coordinate transformation equations are not explicit functions of time.
  2. The potential energy is not an explicit function of the generalized velocities.

- Obtain the equations (and integrals) of motions via the canonical equations of motion:

$$\dot{p}_j = -\frac{\partial H}{\partial q_j}$$

$$\dot{q}_j = \frac{\partial H}{\partial p_j}$$ Take the time derivative of this result and plug into above.

When is H conserved? When is E conserved?

If $H=E$ (see *), then if $\frac{\partial H}{\partial t} = 0$ then both H and E are conserved. *NOTE*: $\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$

If $H \neq E$, then if $\frac{\partial H}{\partial t} = 0$, H is conserved. If $\frac{dE}{dt} = 0$, then E is conserved.

[‡]Note that this is only necessary if there exist external forces of constraint that are doing work on the system which can not be derived from a potential or a "generalized potential". In this case, constraints must be applied prior to the calculation of the conjugate momenta otherwise the momenta will not be complete due to the missing potential terms.