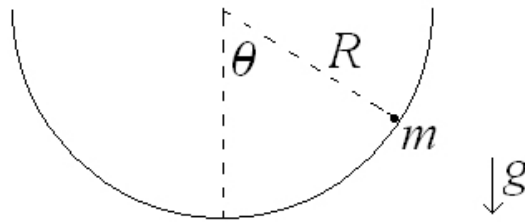


Homework #3 – DUE MONDAY, SEPTEMBER 26, 2007

1. T&M, 3.1
2. T&M, 3.2
3. T&M, 3.3
4. T&M, 3.6
5. T&M, 3.7
6. A point mass, m , is confined to move in 2-dimensions along the inside of a spherically-shaped, frictionless bowl of radius, R . Once given an initial angle of deflection, θ_0 , it is released and allowed to oscillate back and forth, passing through the point at the bottom of the bowl each time.



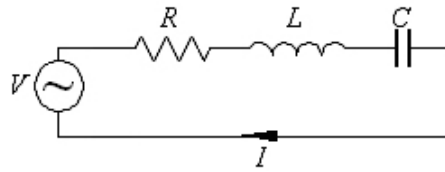
- (a.) Write down the restoring force. (Treat this like a pendulum using polar coordinates – see class notes).
 - (b.) Write down the characteristic frequency, ω_0 , and period of oscillations, τ_0 .
 - (c.) Write down the solution, $\theta(t)$, in terms of ω_0 and θ_0 (assume “small oscillations” such that $\sin\theta \approx \theta$ and neglect the phase shift term in the solutions).
 - (d.) Write down the kinetic and potential energy of the system.
 - (e.) Write down the angular velocity, $\dot{\theta}(t)$.
 - (f.) Determine the time average values of kinetic energy, $\langle T \rangle$, and potential energy, $\langle U \rangle$.
 - (g.) Determine the space average values of kinetic energy, \bar{T} , and potential energy, \bar{U} .
7. Now, assume that the mass in problem (6.) is immersed into a viscous medium with a retarding force:

$$|\vec{F}_r| = \alpha \dot{\theta}$$

acting in the same direction as the restoring force.

- (a.) Write down the equation of motion of the form of equation (3.35) - again, assume “small oscillations” such that $\sin\theta \approx \theta$.
- (b.) Show that, for the system to be critically damped, $\alpha = 2m\sqrt{gR}$.
- (c.) Write down the solution, $\theta(t)$, in terms of ω_0 and θ_0 . NOTE: Since it’s critically damped, you know the solution will be of the form of equation (3.43).

8. Consider the following *RLC* circuit:



The voltage source is $V = E_0 \sin(\omega t)$, and the voltage sum equation is:

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = E_0 \sin(\omega t)$$

- Find the expression of the current, I .
 - Find the voltage across the inductor, V_L .
 - Find the driving frequency, ω_{max} , which makes V_L a maximum.
 - Find the charge resonant frequency, ω_R .
9. A small sphere of mass, m , moves with a constant velocity, \dot{x} , in one dimension. It then collides with a flat, rubber sheet under tension with its surface normal to the trajectory of the sphere. The sheet acts like a “lossy” spring, with a total force composed of both restorative and resistive terms,

$$\vec{F}_{TOTAL} = -2m\alpha(\dot{x} + \alpha x)$$

where α is a constant. (Neglect gravity and all other external forces).

- Show explicitly that the motion is underdamped (find and compare β^2 and ω_0^2) and find the angular frequency of damped oscillations, ω_1 .
- The sphere does not adhere to the sheet and is shot backwards with a new, reduced speed after one single bounce (one-half period). Find the new speed of the sphere and the amount of energy lost from the system due to the collision in terms of α and the unknown amplitude of oscillation, A (from the general solution).

HINT: Use the general solution for underdamped oscillations and use the fact that the “final” kinetic energy is found at $t = \frac{1}{2}\tau_1$ and the “initial” kinetic energy is found at $t=0$.