PHY 3010 – Fall 2007
Homework Set #5 – Due, Wednesday, October 24, 2007

NOTE: HW #6 will be assigned on Monday, October 22, so get started on this set now.

1. Consider the function

$$f = \left(\frac{dy}{dx}\right)^2$$

where $y(x) = x$. The neighboring functions are:

$$\eta(x) = a_n \cos(k_n x)$$

where n is an integer ranging from 0 to infinity and k_n is the wavenumber (unique for each value of n). So here,

$$y(\alpha, x) = y(0, x) + \alpha\eta(x) = x + \alpha a_n \cos(k_n x).$$

Find $J(\alpha)$ between the limits $x = -\frac{n\pi}{k_n}$ and $x = +\frac{n\pi}{k_n}$. Show that $J(\alpha)$ is greater than $J(0)$ for all values of α and that equation (6.4) is satisfied.

HINT #1: Use example 6.1 as a guide.
HINT #2: You can use: $\int \sin^2(ax)dx = \frac{1}{2}x - \frac{1}{4a}\sin(2ax)$
NOTE: You have just shown that the shortest distance between two points is a straight line. In general, this is not a trivial statement – consider if you are no longer confined to Euclidean space (e.g. see examples 6.3 and 6.4). Amazingly, quantum mechanical systems violate the classical solution that you have just shown in the sense that there is now a non-zero probability that a particle may take a path other than a straight line until, of course, you observe it (e.g., apply the position operator to the wave function) and collapse the wave function (the probability distribution) into a selected path. FOR THOSE FAMILIAR WITH FOURIER TRANSFORMS: The method above employs a single component of a Fourier series. To employ the entire series, which you would do in the development of the Feynman path integral formulation, you would just do an infinite sum (or an integral) over n and use the orthogonality of sinusoids to reduce the solution to a similar form.

For problems 2-4, in which you are working examples in the book, you must show explicitly:

- The kinetic and potential energy terms
- The Lagrangian
- The derivation of the Lagrange's equations of motion (for each applicable coordinate).
- The computation of the number of degrees of freedom.

2. Work out example 7.3 in T&M, in
    a. Cartesian coordinates
    b. Polar coordinates..
3. Work out example 7.5 in T&M.
4. Work out example 7.8 in T&M.

5. T&M, 7.4.
6. Using the setup from T&M 7.6, find the Lagrange equations of motion and the integrals of motion ONLY FOR THE CASE IN WHICH THE PLANE CAN NOT SLIDE ALONG THE HORIZONTAL SURFACE.
7. Using the setup from T&M 7.10, find the Lagrange equations of motion and the integrals of motion ONLY FOR THE CASE IN WHICH THE STRING IS MASSLESS.
8. T&M, 7.14.