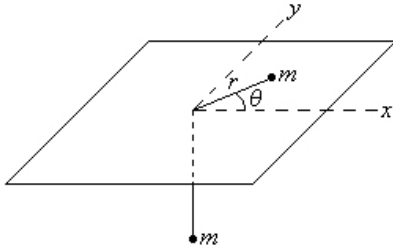


**PHY 3010 – Fall 2007**  
**Homework Set #7**  
**DUE: Friday, 11/9/2007**

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1. Two point particles of equal mass  $m$  are connected via a massless string of length  $d$  (the string can not stretch or buckle). One mass ( $m_1=m$ ) hangs from the string and moves only in a downward direction (it doesn't swing). The string is threaded through a hole in a horizontal, smooth table and is connected to the second mass ( $m_2=m$ ) which is free to move in two dimensions along the table. Define the distance from the hole to the second mass as the  $r$  (e.g. define your origin at the hole). When the system is released, mass  $m_1$  accelerates downward due to gravity, pulling mass  $m_2$  along. Note that mass  $m_2$  can be given any initial angular velocity relative to the fixed table.



The potential energy of the system is given as:

$$U = -mg(d - r)$$

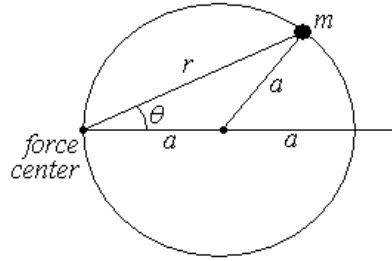
- (a.) Write down the LaGrangian,  $L$ , of the system. (Simplify where possible.) Clearly identify the number of degrees of freedom AND your generalized coordinates. *NOTE: You're not finding forces of constraint so apply any constraint equations immediately to  $L$ .*
- (b.) Using the Lagrange's equations of motion derived from  $L$  in part (a.), define the angular momentum,  $l$ , of mass  $m_2$  and show that it is conserved (show it explicitly – don't just state which coordinates are cyclic).
- (c.) Using your expression from part (b.), write  $l$  as a function of only  $m$ ,  $g$ , and  $r$  required for the system to not translate (e.g.  $\dot{r} = \ddot{r} = 0$ ) AND show that the frequency of rotation is given as  $\omega = \sqrt{g/r}$ .

*CONCEPTUAL HELP: We're looking for the case in which the centripetal force due to the rotation of  $m_2$  balances the gravitational force applied to  $m_1$ .*

Now, let's think in 2-dimensions and just concern ourselves with the motion of  $m_2$  subject to a central force (force center at the hole in the table). To properly transform our original problem into this system, we need to represent our system of masses via a reduced mass,  $\mu$ , and we need to derive a potential function acting on  $\mu$ .

- (d.) Define a reduced mass,  $\mu$ , and a new potential for this system in terms of  $\mu$ .
- (e.) Again, find the frequency of rotation of  $m_2$  necessary for  $m_2$  to not translate in the radial direction – compare it to your answer from part (c.).
2. T&M, 8.18. Use equations 8.21 and 8.41.

3. T&M, 8.11. Use the following diagram as a guide. Define  $r$  as a function of  $a$  and  $\theta$  and apply equation 8.21. Also, solve for the force constant  $k$  in terms of  $l$  and  $\mu$ .



4. T&M, 8.14. Use equation 8.17 to obtain a functional form for  $\left(\frac{dr}{d\theta}\right)^2$  and equate it to the same expression evaluated from the given orbit equation. Solve for  $U$  then the force.
5. T&M, 8.17. Start from equation 8.44 and realize that, from the definition of angular momentum,  $\dot{\theta}_{\min} = \omega_{\min} = \frac{l}{\mu r_{\max}^2}$ .
6. T&M, 8.9. Again use the Virial Theorem and assume that the rocket firing does not change its potential energy (the change in distance from the center of the Earth is negligible).
7. T&M, 8.5.
8. Two masses, separated by a distance,  $r(t)$ , are interacting via a central force with a potential of

$$U(r) = -\frac{k}{r^3} \quad \text{where } k > 0 \text{ is a constant,}$$

resulting in a mutual orbit that can be easily described via a reduced mass,  $\mu$ , as discussed in class.

- (a.) Write down the effective potential,  $V(r)$ , which is plotted on the right. (NOTE: the centripetal component is the same here as for inverse-square potentials.)
- (b.) Find the value of  $r_0$  where the effective potential peaks and find the corresponding value of  $V(r_0)$ . (Find these in terms of  $l$ ,  $\mu$ , and  $k$ .)

