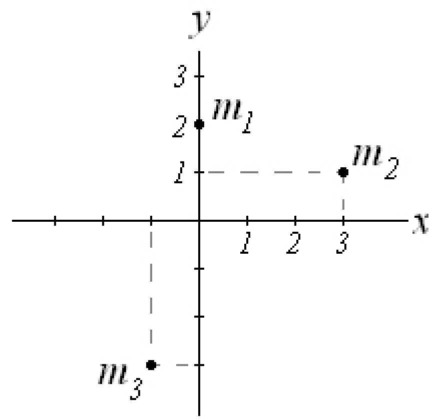
PHY 3010 Fall 2007, HW#9 - LAST ONE!
Due Friday, November 30

1. Three points masses m_1=m, m_2=2m, m_3=m, are located at (0,2,0), (2,1,0), and (-1,-3,0), respectively as shown. Find the inertia tensor for this system of masses. *NOTE: The mass locations are fixed and can not change.*



To get you started, here are the first two matrix elements, I_{11} and I_{12}.

$$I_{ij} = \sum_{\alpha} m_{\alpha} \left[\delta_{ij} \sum_{k} x_{\alpha,k}^2 - x_{\alpha,i} x_{a,j} \right]$$

Note: for m_1, $x_{11}=x_1$, $x_{12}=y_1$, $x_{13}=z_1$.

i=1, j=1:

$$I_{11} = m_1\left[x_1^2 + y_1^2 + z_1^2 - x_1^2\right] + m_2\left[x_2^2 + y_2^2 + z_2^2 - x_2^2\right] + m_3\left[x_3^2 + y_3^2 + z_3^2 - x_3^2\right]$$
$$I_{11} = m\left[y_1^2 + z_1^2\right] + 2m\left[y_2^2 + z_2^2\right] + m\left[y_3^2 + z_3^2\right]$$
$$I_{11} = m\left[(2)^2 + (0)^2\right] + 2m\left[(1)^2 + (0)^2\right] + m\left[(-3)^2 + (0)^2\right] = 4m + 2m + 9m = 15m$$

i=1, j=2:

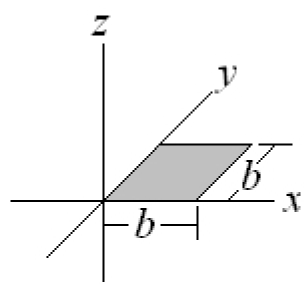
$$I_{12} = m_1\left[0 - x_1 y_1\right] + m_2\left[0 - x_2 y_2\right] + m_3\left[0 - x_3 y_3\right]$$
$$I_{12} = m\left[-x_1 y_1\right] + 2m\left[-x_2 y_2\right] + m\left[-x_3 y_3\right]$$
$$I_{12} = m\left[-(0)(2)\right] + 2m\left[-(3)(1)\right] + m\left[-(-1)(-3)\right] = 0m - 6m - 3m = -9m$$

2. Thornton, problem 11-1.

3. Thornton, problem 11-7. Note, only the component of the force along x does any work. Use F_x=-*kx*. Find *T* and *U* to construct *L* and solve via LEMs.

4. Thornton, problem 11-20. Note, Use conservation of energy.

5. A thin square sheet with sides of length b and with a homogeneous surface mass density, σ, rests in the x-y plane with the origin located on one of its corners.



a. What is the inertia tensor of the square?
b. What are the principal moments of inertia?
c. What are the three principal axis of rotation?
d. Using the parallel-axis theorem, find the inertia tensor for the square in a coordinate system with the origin location at the center of the square ($x=y=b/2$).