Simple Power Law for Transport Ratio with Bimodal Distribution of Coarse Sediment

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Motivation

Objective:

• Develop simple formula for predicting partial transport rates with bimodal mixtures of coarse sediment under waves.

Potential Impact:

• Predicting spatial and temporal evolution of littoral seabed by grain size is important to wave/circulation modeling.
Discrete Particle Model

Model Interactions:
- Grain-grain: elastic-plastic theory & experiment
- Grain-fluid: buoyancy, drag, virtual mass
- Fluid-fluid: eddy viscosity model

(Drake and Calantoni, 2001)
Particle-Particle Interactions

Normal Force (e.g., Walton and Braun, 1986):

loading: \[ F_n = k_1a \]

unloading: \[ F_n = \max \left[ k_2 \left( a - a_0 \right), k_3a \right] \]

coefficient of restitution: \[ e = \left( \frac{k_1}{k_2} \right)^{1/2} \]

Tangential Force (Drake and Walton, 1995):

\[ F_t = \min \left[ |k_t ds|, |\mu F_n| \right] \]
Equation for Translational Particle Motion

\[
\rho_s V_s \frac{d\tilde{u}_s}{dt} = (\rho_s - \rho) V_s \ddot{g} \quad \text{(buoyancy)}
\]

\[
+ \frac{1}{2} \rho C_D^* A |\tilde{u}_f - \tilde{u}_s| (\tilde{u}_f - \tilde{u}_s) \quad \text{(drag)}
\]

\[
+ \rho V_s c_m \left( \frac{D\tilde{u}_f}{Dt} - \frac{d\tilde{u}_s}{dt} \right) \quad \text{(added-mass)}
\]

\[
+ \rho V_s \frac{D\tilde{u}_f}{Dt} \bigg|_{\zeta = \infty} \quad \text{(horizontal pressure)}
\]

\[
+ F_\Phi \quad \text{(inter-particle)}
\]

(e.g., Madsen, 1991) *(e.g., Richardson and Zaki, 1954)
**Bimodal Simulation Suite**

**Particle Diameter Ratios:**

| $D_L/D_S$ | 5/4 | 3/2 | 2/1 |

LARGE particle diameter fixed at 1.5 mm.

**Bed Composition:**

Mass ratio % of particles in simulation domain, $M_L/M_S$.

| 10/90 | 20/80 | 30/70 | 40/60 | 50/50 | 60/40 | 70/30 | 80/20 | 90/10 |

**Monochromatic Waves (6 s period):**

<table>
<thead>
<tr>
<th>velocity skewness;</th>
<th>velocity amplitude (m/s)</th>
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</thead>
<tbody>
<tr>
<td>1.2; 0.85</td>
<td>1.2; 1.10</td>
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<tr>
<td>1.2; 1.10</td>
<td>1.2; 1.35</td>
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243 total different simulations $\rightarrow$ 15,000 CPU hours (CRAY X1)
Power Law Hypothesis

\[
\frac{q_L}{q_S} = K \left( \frac{M_L}{M_S} \right)^{\frac{D_L}{D_S}}
\]

\[
\frac{\bar{v}_L}{\bar{v}_S} = K \left( \frac{M_L}{M_S} \right)^\alpha
\]

- Transport Ratio obeys power law that is independent of forcing.
- Power law contains mass on both sides of the equation → may produce artificially high correlations (Puleo et al., 2005).
- Rewrite power law before performing regressions.
\[ \alpha = \frac{D_L}{D_S} - 1 \]
to within the 95% confidence interval.

\[ K = 4.3 \]
Predicting Partial Transport Rates

\[
\frac{q_L}{q_S} = K \left( \frac{M_L}{M_S} \right)^{\frac{D_L}{D_S}}
\]

\[
q = q_L + q_S
\]

\[
q = \begin{cases} 
  k \langle u^3 \rangle + K_a \left( a_{\text{spike}} - a_{\text{crit}} \right) & \text{for } a_{\text{spike}} > a_{\text{crit}} \\
  k \langle u^3 \rangle & \text{for } a_{\text{spike}} \leq a_{\text{crit}}
\end{cases}
\]

\[
a_{\text{spike}} = \frac{\langle a^3 \rangle}{\langle a^2 \rangle} \quad k = 0.8 \text{ kg s}^2 \text{ m}^{-4}
\]

\[
K_a = 0.07 \text{ kg s m}^{-2} \quad a_{\text{crit}} = 1 \text{ m s}^{-2}
\]

(Drake and Calantoni, 2001)
Predicted Versus Measured Partial Transport Rates
Summary

\[ \frac{q_L}{q_S} = K \left( \frac{M_L}{M_S} \right)^{\frac{D_L}{D_S}} \]

Caveats

• Grains must sort vertically!
• Practical use in morphodynamic model requires sophisticated bed tracking scheme (a lot of book keeping!)
Future Directions

• Physical/theoretical basis for the power law?
• Steady flow simulations
• Simulations of dry granular flows
• More complex nearshore flows and finer grains (i.e., flows with turbulent lift and wave-current interaction)